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Research article

MATHEMATICAL MODEL TO ONITOR SEEPAGE AND DISCHARGE OF NONLINEAR FLOW NET INFLUENCED BY DEGREE OF POROSITY AND VOID RATIO IN HETEROGENEOUS GRAVEL FORMATION

Eluozo, S. N[1], Nwaoburu A .O[2]

[1]Subaka Nigeria Limited, Port Harcourt, Rivers State of Nigeria
[1]Director & Principal Consultant, Civil & Environmental Engineering, Research & Development
E-mail: Soloeluozo2013@hotmail.com
[2]Department of Mathematics/Computer Science, Faculty of Sciences, Rivers State University of Science and Technology, Nkpolu, Port Harcourt.
E-mail:nwaoburu.adols@ust.edu.ng

Abstract

Monitoring the seepage force in deltaic formation is to determine the extent to which nonlinear flow net are influences by porosity, such direction of flow influences stratification of deposited structures, void ratio percentage are some of the formation characteristic that influences seepage forces and flow net, these are base on the assessment of formation characteristics that identified porosity predominance to have pressure in the formation. The behaviuor of the seepage are base on the head loss as water flows through soils formation, there are the tendencies of a reaction against the soil grains, and they are determined by the strength of cohesion less soils produced by pressure between the grains, water flowing downward through fine grained soils increases the strength of such soils. There no doubt that when water flows upward through the soil, there is tendency of gravitational pressure between the grains, this condition is reduced by the seepage forces including the strength of the soil. These engineering properties at these condition develop challenges in construction of hydraulic structures, the study establish the variation influences of nonlinear flow net condition in deltaic formation, The study has precisely express the need for Seepage water through soil becoming an important phenomenon to be considered while designing different water retaining structure, like bridges etc. the effect of seepage on the stability of earth slopes is of immense importance for the design of earth slopes & foundations.

Keywords: mathematical model, seepage, nonlinear flow net and heterogeneous formation.

1. Introduction

Among hydro-geological, geological, and geotechnical problems in earth dams, seepage is a major concern associated with dams. Because it threatens dam stability and may cause unforeseen failure (Close et al., 2002 Saeed et al 2013). Many methods such as using of cut off wall and injection to avoid seepage are proposed. Otherwise the prediction of seepage discharge through earth dam can be useful for seepage controls. Fealeh Khaseh dam located in Zanjan province, west of Iran, is used as a case study in this paper. Although there are many established theoretical relation between seepage and soil properties their association and evaluation of seepage, requires system identification techniques. The interdependency of factors involved in such problems prevents the use of regression analysis and demands a more extensive and sophisticated method. The Group Method of Data Handling (GMDH) type of artificial neural networks (ANN) optimized by Genetic Algorithms (GAs) can be used for complex systems modeling, where unknown relationships exist between the variables, without having a specific knowledge of the processes. In recent years, the use of such self-organizing neural networks has led to successful application of the GMDH-type of algorithm in geotechnical sciences (Ardalan et al., 2009; Kalantary et al., 2009).

The zoned type earth dams are formed by different zones constructed of especial material; furthermore, each zone has effect on stability and operation of earth dams. Because in zoned earth dams different particle with various size are settled beside each other, water infiltrate among them. This situation continues gradually until difference between water potential in up and down stream cause moving particles from porosity and starting piping suddenly. This event is named as seepage. Lakshmi (2003) in theory, seepage is named gravitation water which flow into soil particles porosity in result of gravity. Seepage in dam is similar to flow in an open channel because their free surface and atmosphere pressure. But, underground water is including two sections liquid/solid; and, flow velocity and discharge are controlled by soil permeability. A method to display flow in body and foundation of earth dam is Flow net. The primary line in flow net is named as phreatic line which restricts seepage. In spite of existing different permeability in soils, when the ratio of horizontal to vertical permeability is same, the phreatic line will reach identical position eventually. Kalkani, (1989). with pay attention to this note in zoned earth dams that permeability in shell is more than core. When this difference become very much, shall is ignored to calculate the seepage rate and core is assumed as a homogeneous dam, Goldin et al (1992).

2. Theoretical background

The behaviour of seepage in soil is base on several conditions through its depositions of the formation, the behaviour of seepage are reflected on the rate of groundwater deposition between the intercedes of the formation, the availability of groundwater has negative and positive impact in human settlement, in some construction project, the deposition of groundwater develop lots of challenges, seepage generate three major types of problems, the mostly challenging one is loose of stored water through earth dam or foundation, including instability of slopes and foundation on hydraulic structures, these are base on the forces exerted by percolating water, and another is settlement of structures on founded above compressible layers due to expulsion of water from high deposited void ratio, this is as a result of load application, it is has been noted that when water flow through the intercedes of formations, the water head will always dissipated through viscous friction, these are during energy dissipation whereby a drag forces is exerted on soil particles through a direction of flows. Further more in construction period If the level of the water on the excavation side of the wall is always the same as the phreatic surface on the soil side, there will be no water flow and hydrostatic conditions (without unbalanced hydrostatic forces on the wall) that will take place. However, there are many conditions that make this impossible, including tidal variations and pumping braced excavations dry below the prevailing water table. In cases such as these, water movements in the soil will take place and these will have a variety of effects on soil and wall alike. More so The coefficient of permeability, k, is defined as the rate of discharge of water at a temperature of 20° C under conditions of laminar flow through a unit cross-sectional area of a soil medium under a unit hydraulic gradient. The coefficient of permeability has the dimensions of velocity and is usually expressed in centimeters/second. The permeability of a soil depends primarily on the size and shape of the soil grains, the void ratio of the soil, the shape and arrangement of the voids, and the degree of saturation. Permeability computed based on Darcy's law is limited to the conditions of laminar flow and complete saturation of the voids. Flow of water through a soil mass results in some force being exerted on the soil itself. To evaluate the seepage force per unit volume of soil, this implies that consideration of a soil mass bounded by two flow lines. More so, Flow lines are the average flow path of a particle of water flowing from the upstream to the downstream. Similarly, representation from the energy of flow lines may be equal to the potential called the equipotential lines, along any equipotential line. The energy available to cause such flow is the same. The network of flow lines and equipotential lines is called a flow net, a concept that illustrates graphically by experts show how the head or energy is lost as water flows through a porous medium. A flow net is actually a graphical solution of Laplace's equation in two dimensions. More so a seepage tank is used to demonstrate the pattern of fluid flow around a sheet-pile wall in soil engineering. A dye injected on the upstream surface of the sheet-pile wall demonstrates the flow lines. Several piezometers are placed upstream and downstream of the sheet pile to measure the total head [sujoy, 2010].

3. Developed governing equation

$$\frac{Av + As}{Av}\,\frac{\partial \Phi s}{\partial L} - \frac{\partial \Phi^2 s}{\partial t} + \frac{(Av + As)}{AvL} + \frac{\partial \Phi s}{\partial L}\,\Phi\,\frac{(Vv + Vs)}{Vv} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \qquad (1)$$

The developed governing equation express the parameters the definitely establish the pressure of flow net under the influences of seepage pressures in design and construction of hydraulic structures, the behaviour of the system are determined by the rate of porosity as these condition are predominant parameter in the study location that pressure the level of seepage developing nonlinear flow net in the formation. lots of geotechnical engineers has definitely express lots of challenges experiences in the design and construction of hydraulic structural foundation, these conditions has express various difficulties in the construction industries .

$$\frac{\partial \Phi s}{\partial L} = S^1 \Phi s(L) - S\Phi s(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\partial \Phi s}{\partial t} = S^1 \Phi s(L) - S\Phi s(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\frac{\partial \Phi s}{\partial L} = S^1 \Phi s(L) - S\Phi s(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

$$S^1 V\Phi s(L) - \frac{Av + As}{Av}\left[S^1(L) - S^1\Phi s(o)\right] + V\frac{(Av + As)}{AvL}\left[S^1\Phi s(t) - S^1\Phi s(z)\right] \ldots \quad (5)$$

$$S^1(L) - \Phi s(o) = \frac{Av + As}{Av} S^1 \Phi s(o) - \frac{(Av + As)}{AvL}\Phi s(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

$$\Phi \frac{(Av + As)}{AvL} \Phi s(L) - \Phi \frac{(Av + As)}{AvL} S\Phi s(o) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$\Phi \frac{(Vv + Vs)}{Vv} \Phi s(z) - S\Phi s(o) \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

The rate of porosity are the most predominant influence in the study location, precisely in study, there need to express the behaviour of porosity relationship with volume of void under this derived solution , this is to ensure that despite porosity predominant monitored, the rate of its degree are base on some high percentage of void deposition in the formation, the derived solution applying this mathematical approach to ensure that the relation in the system are thoroughly correlated, these approach definitely developed some boundary limited stated below.

Let $\Phi s(o) = 0$

We have

$$S^1(L) - \frac{(Av + As)}{Av}\Phi s(o) + \Phi \frac{(Av + As)}{AvL}\Phi s(t) + \Phi \frac{(Vv + Vs)}{Vv} S^1 \Phi s(L) \quad \ldots \quad (9)$$

$$\Phi S^1(L) = \frac{1}{S}\left[\frac{(Av + As)}{Av}\Phi s(L) - \Phi \frac{(Av + As)}{AvL} S^1 + \Phi \frac{(Vv + Vs)}{Vv} S^1 \Phi s(L)\right] \quad \ldots \quad (10)$$

$$\Phi S(L) = \frac{1}{S^1}\left[\frac{(Av + As)}{Av}\Phi s(L) - \Phi \frac{(Av + As)}{AvL} S^1 + \Phi \frac{(Vv + Vs)}{Vv}\right] \quad \ldots\ldots\ldots\ldots \quad (11)$$

$$\Phi S^1(L) = \frac{\frac{(Av + As)}{Av}\Phi s(L) - \Phi \frac{(Av + As)}{AvL}\Phi s(t) + \Phi \frac{(Vv + Vs)}{Vv}}{S^1} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

$$\Phi S(L) = \frac{(Av + As)}{Av} S^1 \Phi s(L) + \Phi \frac{(Vv + Vs)}{Vv}\Phi S(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$\Phi S(L) = \frac{\frac{(Av + As)}{Av}\Phi s(L) = \Phi \frac{(Av + As)}{AvL}\Phi s(L) + \Phi \frac{(Vv + Vs)}{Vv}}{S} \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$\Phi S(L) = \left[\frac{(Av + As)}{Av} + \Phi \frac{(Av + As)}{AvL} + \Phi \frac{(Vv + Vs)S^1}{Vs}\right]\Phi S(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

$$S^1\Phi S(L) = \left(\frac{(Av + As)}{Av} + \Phi \frac{(Av + As)}{AvL} + \Phi \frac{(Vv + Vs)S^1}{Vs}\right)\Phi S(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$\frac{Av + As}{Av} + \Phi \frac{(Av + As)}{AvL} + \Phi \frac{(Vv + Vs)S^1}{Vs} \quad \ldots\ldots\ldots\ldots\ldots \quad (17)$$

$$\Phi S(L) \frac{S^1(L)}{\frac{Av + As}{Av} + \Phi \frac{(Av + As)}{AvL} + \Phi \frac{(Vv + Vs)S^1}{Vs}} \quad \ldots\ldots\ldots\ldots\ldots \quad (18)$$

The derived solutions thoroughly express the similarities functions of these parameters that pressure the seepage force in the formation. Such condition are established in the system to defined there various functions and limits, the system with their various limits express their reaction with other parameters in the system to defined there various angle that they express the power under the influences of stratification in the formation, because the variation of the stratification will definitely express variation in flow net, the expression of seepage in construction project are through several defined variation of the lithology of the formation, these condition are paramount in deltaic area, such condition in these deltaic formation will definitely define those condition in the development of seepage flow in the soil.

Furthermore, considering the boundary condition, we have the following

$$\text{At } t = 0 \qquad \Phi^1 S(o) = \Phi S(o) = 0$$

$$\frac{Av + As}{Av}\Phi S(L) - \Phi \frac{(Av + As)S^1}{AvL}\Phi S(L) + \Phi \frac{(Vv + Vs)}{Vv}\Phi S(t) = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (19)$$

$$\Phi S(L) = \frac{Av + As}{Av} - \Phi \frac{(Av + As)}{AvL} + \Phi \frac{(Vv + Vs)}{Vs} \quad \ldots\ldots\ldots\ldots\ldots \quad (20)$$

Considering the following boundary conditions when

$$\text{At } t > 0 \qquad \Phi^1 S(o) = \Phi S(o)$$

Applying the boundary condition into this equation

$$\frac{Av + As}{Av}\Phi S(L) - \frac{(Av + As)}{Av} + \Phi \frac{(Av + As)}{AvL}\Phi S(L) + \Phi \frac{(Av + As)}{AvL}\Phi So -$$

$$S(L) + \Phi \frac{(Vv + Vs)}{Vv} VS(t) + \Phi \frac{(Vv + Vs)}{Vv} VSo + S(t) \quad \ldots\ldots\ldots\ldots\ldots \quad (21)$$

$$\frac{(Av+As)}{Av}(L)-\frac{(Av+As)}{AvL}\Phi S(o)\ \frac{(Av+As)}{Av}\Phi So\ +\Phi\frac{(Vv+Vs)}{Vv}\Phi So \quad \ldots \quad (22)$$

$$\Phi S(L)=\left[\frac{Av+Ass}{Av}=\Phi\frac{(Av+As)}{Av}+\Phi\frac{(Av+As)}{AvL}+\Phi\frac{(Vv+Vs)}{Vv}\right]\Phi So \quad \ldots \quad (23)$$

$$\Phi S(L)=\frac{Av+Ass}{Av}-\frac{(Av+As)}{Av}-\Phi\frac{(Av+As)}{AvL}+\Phi\frac{(Vv+Vs)}{Vv}\Phi So \quad \ldots\ldots\ldots (24)$$

$$\Phi S(L)=\frac{\frac{(Av+Ass)}{Av}-\frac{(Av+As)}{Av}-\Phi\frac{(Av+As)}{AvL}+\Phi\frac{(Vv+Vs)}{Vv}\Phi So}{\frac{(Av+As)}{Av}S-\frac{(Av+As)}{Av}+\Phi\frac{(Vv+Vs)}{Vv}} \quad \ldots\ldots\ldots (25)$$

Further expression are defined in this condition stated above, the derived solution express several other ways in the system that pressure the formation variation through the predominant deposition of porosity expressed, the derived solution establish the pressure from the volume of void with respect to various limits of effective stress that are found in soil through the rate of unconsolidation of the strata, various direction of flow in the formation are thoroughly monitored, the derived solution expresses various dimensions of influences from those parameters, the system in this stage will definitely monitor the behaviour of the formation as it is expected to generate various flow condition depending of the rate of unconsolidation of the deltaic formation.

Applying quadratic equation to determine denominator for the equation

$$\frac{Av+Ass}{Av}-\Phi\frac{(Av+As)}{AvL}+\Phi^2\frac{(Vv+Vs)}{Vv}=0 \quad \ldots\ldots\ldots(26)$$

$$s=\frac{-b\pm\sqrt{b^2-4ac}}{2ac} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (27)$$

Where $a=\frac{Av+As}{Av}$, $b=\Phi^2\frac{(Av+As)}{AvL}$ and $c=\Phi\frac{(Vv+Vs)}{Vv}\Phi S$

For simplicity denoting the expressed functions parameter of the following

Let $\frac{Av+As}{Av}=Q$, $\Phi\frac{(Av+As)}{AvL}=\lambda^2$ and $\Phi\frac{(Vv+Vs)}{Vv}=\alpha$

Integrating the express parameters into the quadratic function we have:

$$S=\frac{-\lambda^2\pm\sqrt{\lambda^2-4Q\alpha}}{2\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (28)$$

$$\left[S_1=\frac{\lambda^2+\sqrt{\lambda^2-4Q\alpha}}{2\alpha}\right]\left[S_2=\frac{\lambda^2-\sqrt{\lambda^2-4Q\alpha}}{2\alpha}\right]$$

$$\ell^{\left[\frac{\sqrt{\lambda^2-4Q\alpha}}{2Q\alpha}\right]t\left[\frac{\lambda\sqrt{\lambda^2-4Q\alpha}}{2\alpha}\right]t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (29)$$

The inverse Laplace of the equation yield

$$VS(L)=\left[\frac{Q}{t}+Q+\lambda+\alpha\right]\Phi So\;\ell^{\left[\frac{\lambda+\sqrt{\lambda^2+4Q\alpha}}{2\alpha}\right]t}$$

$$\left[\left[\frac{\lambda-\sqrt{\lambda^2-4Q\alpha}}{2\alpha}\right]\right]t-\left[\frac{\lambda-\sqrt{\lambda^2+4Q\alpha}}{2\alpha}\right]t \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (30)$$

$$VS(L)=\left[\frac{\lambda}{t^2}\;\Phi So\right]\left[\left[\frac{\lambda+\sqrt{\lambda^2-4Q\alpha}}{2Q\alpha}\right]\right]\ell^{\left[\frac{\lambda\sqrt{\lambda^2-4Q\alpha}}{2\alpha}\right]t}$$

$$\ell^{\left[\frac{\lambda\sqrt{\lambda^2-4Q\alpha}}{2Q\alpha}\right]t-\left[\frac{\lambda\sqrt{\lambda^2-4Q\alpha}}{2Q\alpha}\right]t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (31)$$

At this point $VSo \;=\; 0 \quad t\neq 0$

For equation (30) at $t\;=\;0\;\; VS(o)\;=\;VSo,\; we\;\; have$

$$VSo\;=\left[(Q+\lambda+\alpha)\,VSo\;\;(1+1+1)\right]=0=(Q+\lambda+\alpha)$$

$$Hence\;\; Q+\lambda+\alpha\;=\;0$$

Equation (31) becomes

$$\Phi S(L)-\Phi So\left[\frac{\lambda}{t^2}+2\right]\left[\frac{\lambda+\sqrt{\lambda^2-4Q\alpha}}{2Q\alpha}\right]t\;\left[\frac{\lambda\sqrt{\lambda^2-4Q\alpha}}{2\alpha}\right] \qquad \ldots\ldots\ldots \qquad (32)$$

We recall that $Hence\;\;\ell^{x}+\ell^{-x}\;=\;2Cos\,x,$ so that equation (32) can be expressed as:

$$\boxed{\Phi S(L)-[\lambda+2]\Phi So\;Cos\left[\frac{\lambda\sqrt{\lambda^2-4Q\alpha}}{2\alpha}\right]t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (33)$$

The expression here is the final model that has been developed to monitor seepage force under non linear flow net in the study area. The direction of flow are base on the soil formation, the structural stratification determine the rate of seepage forces in the study area, such condition consider in column of soil whereby the height of water surface in the reservoir is raised, it implies that water pressured at the bottom of the soil will definitely increase, thus the drag force on the soil particles will become greater. Such condition of seepage forces are influences by porosity predominant as these study. Precisely look at high degree of porosity influences on the seepage forces in non linear flow net, the express model will identifies several conditions that may cause developed rapid seepage in deltaic formation.

4. Conclusion

The strength of soil formations and of natural deposits is reliant is not only upon the static characteristics of the soil, but also upon the forces generated by water as it seeps or flows through the micropores of the soil formations. As it supported to its verdict in the design of earth formations including the stabilization of earth depositions, the engineer should be able to approximate thorough analyses of the degree of seepage forces and pressures including the quantities of water flowing through the soil strata. Because the loss head as water flows through soils is a reaction against the soil grains, this implies that the strength of cohesion less soils is produced by pressure between the grains, water flowing downward through fine grained soils increases the strength of such soils. When water flows upward through the soil, gravity pressure between the grains is reduced by the seepage forces and the strength of the soil is reduced. the developed model monitor the condition were porosity are predominant under the influences of deltaic formation through the rate of forces as the water flows through the formations, the water head will normally experiences dissipation, whereby a drag forces is exerted on the soil particles through the direction of flows.

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